**MA 331** **Name (Print):** ______________________

**Fall 2017**

**Exam Review 1**

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**1.)** What is the order of the following differential equations?

a.) $y' = e^(y'') - yy'$

b.) $\frac{d^2y}{dx^2} - 3y^2 = \frac{dy}{dx}$

c.) $(y''')^2 + y' = 3x^2 - y$

**2.)** State whether the following differential equations are linear/nonlinear and separable/non-separable.

a.) $\frac{dy}{dx} = \frac{x^3-2y}{x}$

b.) $y' = t^2 \ln(y)$

c.) $\frac{dy}{dx} = -\frac{2xy+1}{x^2+2y}$

d.) $x\frac{dy}{dx} + xy = 1 - y$

**3.)** Find the integrating factor (DO NOT SOLVE)

a.) $ty' + 2y = \sin(t)$

b.) $2y' = 3t^2 - y$

c.) $ty' + (t+1)y = 1$

**4.)** Solve the following differential equations

a.) $2y' = 3t - y$

b.) $y' = \frac{1-2x}{y}$

c.) $y' = x\frac{x^2+1}{4y^3}$

**5.)** Solve the following initial value problems.

a.) $ty' = t^2 - t + 1 - 2y, \quad y(1) = \frac{1}{2}, \quad t > 0$

b.) $y' = \frac{e^{-x}-e^{x}}{3+4y}, \quad y(0) = 1$

c.) $y' = \frac{3x^2-e^{x}}{2y-5}, \quad y(0) = 1$

**6.)** The population of flies in a lab increases at a rate proportional to the current population. The population of flies doubles once per week. Initially, there are 150 flies. Find and solve the differential equation that describes the population of flies with respect to time.

**7.)** Suppose that a population can be divided into two parts: those who have chicken pox and can infect others and those who do not have it but are susceptible. Let $x$ be the proportion of susceptible individuals and $y$ the proportion of infections individuals (so $x + y = 1$). The disease spreads when infectious individuals interact with susceptible individuals, and the rate of spread $\frac{dy}{dt}$ is proportional to the number of such interactions. Assuming that the susceptible and infectious individuals move about freely so the number of contacts in proportional to the product of $x$ and $y$. Therefore, the equation governing the the proportion of infectious individuals is written by:

$$\frac{dy}{dt} = \alpha yx, \quad y(0) = y_0$$

a.) Show that the differential equation can be written as:

$$\frac{dy}{dt} = \alpha y(1 - y)$$

b.) Find the equilibria and determine their stabilities.

c.) Solve the initial value problem and verify that the conclusion you reached in b) is correct.

**8.)** A second order chemical reaction involve the interaction of one molecule of a substance $P$ with one molecule of a substance $Q$ to produce one molecule of a new substance $X$. Suppose that $p$ and $q$, where $p \neq q$ are the initial concentrations of $P$ and $Q$, respectively, and let $x(t)$ be the concentration of $X$ at time $t$. Then $p - x(t)$ and $q - x(t)$ are the concentrations of $P$ and $Q$ at time $t$, and the rate at which the reaction occurs is given by the equation:

$$\frac{dx}{dt} = \alpha(p - x)(q - x)$$

where $\alpha$ is a positive constant.

a.) Assuming $x(0) = 0$, and $p < q$, determine the limiting value of $x(t)$ as $t \to \infty$.

b.) Solve the initial value problem given in a). Does your answer match?

c.) Now assume that the initial amount of substance $P$ and $Q$ are the same, so $p = q$. Then the differential equation can be written as

$$\frac{dx}{dt} = \alpha(p - x)^2$$

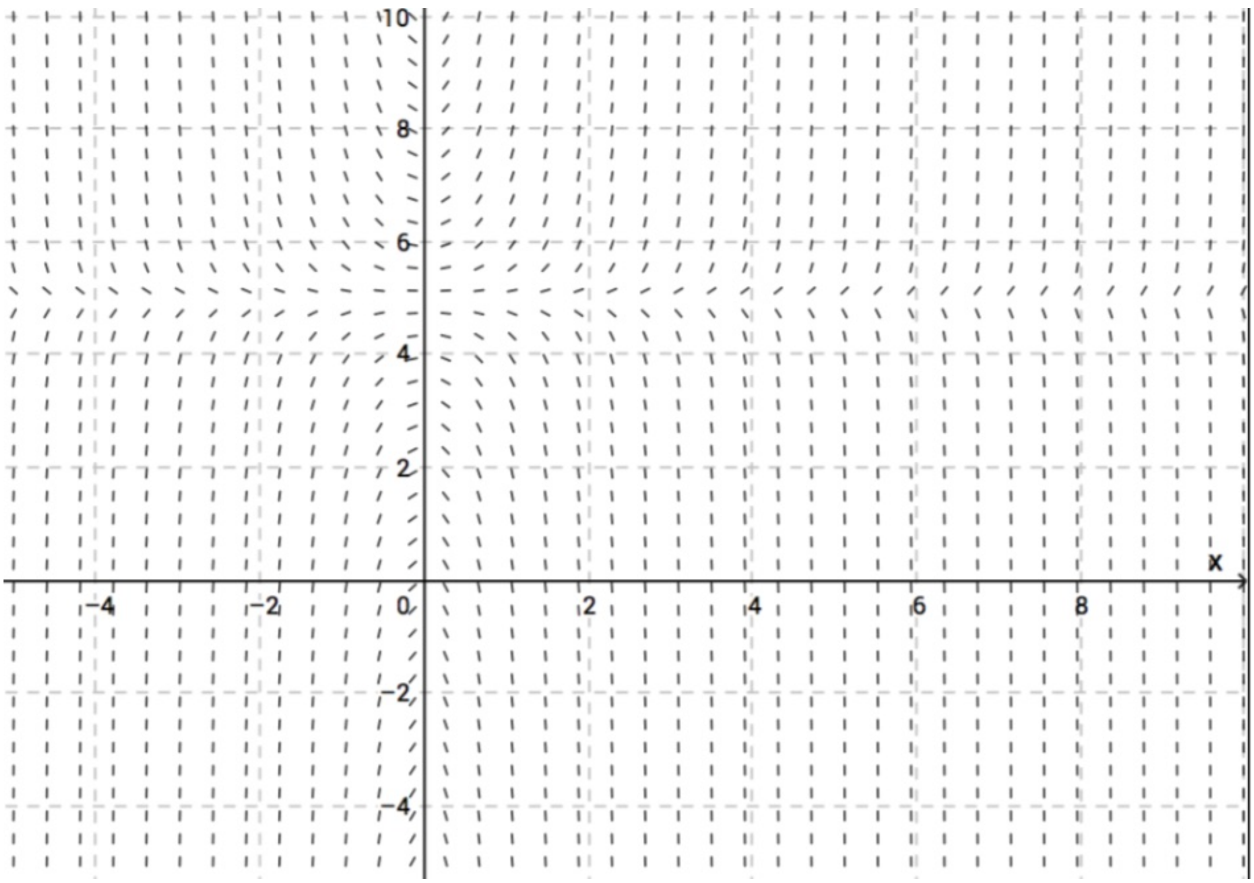
. If x(0)=0, determine the limiting value of $x(t)$ as $t \to \infty$

d.) Solve the initial value problem given in c). Does your answer match?

**9.)** Consider the equation

$$\frac{dy}{dt} = 2t(y-5)$$

. The slope field is given by:




a.) Draw a solutions on the slope field that correspond to the initial condition $y(-2) = 6$ . What is the long-term behavior of the solution?

b.) Draw a solutions on the slope field that correspond to the initial condition $y(0) = 4$. What is the long-term behavior of the solution?

c.) Solve the differential equation for each of the initial conditions given in a) and b).