MA 331 **Name (Print):** ______________________
Fall 2017
Exam Review 2

1.) Find the characteristic equation

a.) $y'' + 2y' - 3y = 0$

b.) $y'' - 2y' + y = 0$

c.) $3y'' - 2y' + 6y = 0$

d.) $-2y'' = 7y' - 3y$

e.) $4y'' = 2y' + 6y$

2.) Find the general solution to the 2nd order differential equation

a.) $y'' + 3y' + 2y = 0$

b.) $6y'' - y' - y = 0$

c.) $y'' - 2y' + 2y = 0$

d.) $y'' - 6y' + 9y = 0$

e.) $4y'' - 4y' - 3y = 0$

f.) $4y'' + 9y = 0$

g.) $4y'' - 9y = 0$

h.) $4y'' - 4y' + y = 0$

3.) Solve the initial value problem

a.) $y'' + y' - 2y = 0, \quad y(0) = 1, \quad y'(0) = 1$

b.) $y'' + 4y' + 3y = 0, \quad y(0) = 2, \quad y'(0) = -1$

c.) $6y'' - 5y' + y = 0, \quad y(0) = 4, \quad y'(0) = 0$

d.) $y'' - 2y' + 5y = 0, \quad y\left(\frac{\pi}{2}\right) = 0, \quad y'\left(\frac{\pi}{2}\right) = 2$

e.) $y'' + 4y' + 5y = 0, \quad y\,(0) = 1, \quad y'\,(0) = 0$

f.) $9y'' - 12y' + 4y = 0, \quad y\,(0) = 2, \quad y'\,(0) = -1$

g.) $9y'' + 6y' + 82y = 0, \quad y\,(0) = -1, \quad y'\,(0) = 2$

h.) $y'' - 6y' + 9y = 0, \quad y\,(0) = 0, \quad y'\,(0) = 2$

i.) $y'' + 3y' = 0, \quad y\,(0) = -2, \quad y'\,(0) = 3$

j.) $y'' + 4y' + 4y = 0, \quad y\,(-1) = 2, \quad y'\,(-1) = 1$

4.) Consider a mass-spring system, given by $x''(t) + \gamma x'(t) + x = 0$

a.) Assume the undamped case, $\gamma = 0$. Solve for $x(t)$. You do NOT need to find C_1 or C_2.

b.) What is the period of the oscillations for your system in part (a)? Recall that the period for $\cos(at) = \frac{2\pi}{a}$

c.) Now assume the underdamped case, $\gamma > 0$ and $\gamma^2 < 4$. Solve for $x(t)$. Your solution should depend on γ. You do NOT need to find C_1 or C_2.

d.) What is the period of oscillations for your system in part (c)?

e.) For what value of γ will the period of your damped oscillator (found in d) be 50% greater than the period of your undamped oscillator (found in b)?

5.) The flow of current in a circuit can be governed by Kirchoff's 2nd law: $LQ''+RQ'+\frac{1}{C}Q = E(t)$ where Q represents the charge in coulombs, R is the resistance (ohms), C is the capacitance (farads), L is the inductance (henrys), and $E(t)$ represents the impressed voltage (volts) as a function of time. $I = \frac{dQ}{dt}$ represents the current (ampres). See the attached figure.

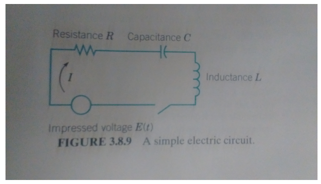

FIGURE 3.8.9 A simple electric circuit.

Assume the series circuit has a capacitor of 10^{-5} farad, a resistor of 3×10^2 ohms, and an inductor of 0.2 henry. The *initial* charge on the capacitor is 10^{-6} coulomb and the *initial* current is 0 ampres. Assume no impressed voltage (i.e., $E(t) = 0$).

a.) Write the initial value problem for $Q(t)$.

b.) Solve the initial value problem to obtain $Q(t)$.

6.) For the following, compute the approximate solution using Euler's method. Recall that Euler's method is given by $t_{n+1} = t_n + h$ and $y_{n+1} = y_n + h * f(t_n, y_n)$ for the differential equation $y' = f(t, y)$. h represents the step size. You do not need to fill in the table.

a.) $y' = t^2 - y$, $y(0) = 1$. Let h=0.5. Find Euler's approximation for $y(2)$

t_i	y_i	$f(t_i, y_i) =$	y_{i+1}

b.) $y' = y + 2$, $y(0) = 0$. Let h=0.5. Find Euler's approximation for $y(2)$

t_i	y_i	$f(t_i, y_i) =$	y_{i+1}

7.) Consider the following matrices:

$$A = \begin{pmatrix} 4 & 0 & 1 \\ 6 & 9 & 2 \\ 8 & 1 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & -1 & 4 \\ -1 & 5 & 1 \\ 7 & 1 & 7 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 3 & 4 \\ -6 & 7 & 3 \end{pmatrix}, \quad D = \begin{pmatrix} 9 & 5 \\ 7 & 8 \\ 1 & 2 \end{pmatrix}, \quad E = \begin{pmatrix} 8 & 1 \\ 2 & 7 \end{pmatrix}, \quad F = \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$$

Find the following quantities:

a.) AB b.) BA c.) CD d.) DC e.) EF

f.) $|E|$ g.) $|F|$ h.) E^{-1} i.) F^{-1}

j.) Eigenvalues and Eigenvectors of E

k.) Eigenvalues and Eigenvectors of F

8.) Find the general solution to the system of equations

a.) $$\begin{aligned} x_1'(t) &= x_1 - 2x_2 \\ x_2'(t) &= 3x_1 - 4x_2 \end{aligned}$$

b.) $$\begin{aligned} x_1'(t) &= x_1 + x_2 \\ x_2'(t) &= 4x_1 + x_2 \end{aligned}$$

c.) $$\begin{aligned} x_1'(t) &= 2x_1 - 5x_2 \\ x_2'(t) &= x_1 - 2x_2 \end{aligned}$$

d.) $$\begin{aligned} x_1'(t) &= 2x_1 - \frac{5}{2}x_2 \\ x_2'(t) &= \frac{9}{2}x_1 - x_2 \end{aligned}$$

e.) $$\begin{aligned} x_1'(t) &= 4x_1 - 2x_2 \\ x_2'(t) &= 8x_1 - 4x_2 \end{aligned}$$

f.) $$\begin{aligned} x_1'(t) &= -\frac{3}{2}x_1 + x_2 \\ x_2'(t) &= -\frac{1}{4}x_1 - \frac{1}{2}x_2 \end{aligned}$$

9.) Solve the initial value problems

a.) $$\begin{aligned} x_1'(t) &= x_1 - 2x_2, & x_1(0) &= 0 \\ x_2'(t) &= 3x_1 - 4x_2, & x_2(0) &= 2 \end{aligned}$$

b.) $$\begin{aligned} x_1'(t) &= x_1 + x_2, & x_1(0) &= 1 \\ x_2'(t) &= 4x_1 - 2x_2, & x_2(0) &= 1 \end{aligned}$$

c.) $$\begin{aligned} x_1'(t) &= -3x_1 + 2x_2, & x_1(0) &= 1 \\ x_2'(t) &= -x_1 - x_2, & x_2(0) &= -2 \end{aligned}$$

d.) $$\begin{aligned} x_1'(t) &= x_1 - x_2, & x_1(0) &= 2 \\ x_2'(t) &= 5x_1 - 3x_2, & x_2(0) &= -1 \end{aligned}$$

e.) $x_1'(t) = -\frac{5}{2}x_1 + \frac{3}{2}x_2, \qquad x_1(0) = 3$

$x_2'(t) = -\frac{3}{2}x_1 + \frac{1}{2}x_2, \qquad x_2(0) = -1$

f.) $x_1'(t) = 7x_1 + x_2, \qquad x_1(0) = 2$

$x_2'(t) = -4x_1 + 3x_2, \qquad x_2(0) = -5$

10.) Consider the Romeo and Juliet problem. For each of the following examples do the following:

- Describe, in words, how Romeo and Juliet react to each other and themselves
- Solve the differential equation governing their relationship
- Discuss their long-term feelings for each other.

a.) $R'(t) = J,\ J'(t) = -R$

b.) $R'(t) = -2R + J,\ J'(t) = R - 2J.$

11.) Do opposites attract? Consider a Romeo and Juliet problem with the following equations $R'(t) = R + J,\ J'(t) = -R - J$

a.) Describe, in words, how Romeo and Juliet react to each other and themselves

b.) Find the general solution to Romeo and Juliets feelings

c.) Find the solution if Romeo initially likes Juliet ($R(0) = 1$) and Juliet initially dislikes Romeo ($J(0) = -2$). What happens to their relationship?

d.) Find the solution if Romeo and Juliet initially like each other, $R(0) = 1$, $J(0) = 1$. What happens to the relationship?