MA 331
Fall 2017
Exam Review 2 Answers

1.) a.) $r^2 + 2r - 3 = 0$

b.) $r^2 - 2r + 1 = 0$

c.) $3r^2 - 2r + 6 = 0$

d.) $-2r^2 - 7r + 3 = 0$

e.) $4r^2 - 2r - 6 = 0$

2.) a.) $y = c_1 e^{-t} + c_2 e^{-2t}$

b.) $y = c_1 e^{\frac{t}{2}} + c_2 e^{-\frac{t}{3}}$

c.) $y = c_1 e^t \cos t + c_2 e^t \sin t$

d.) $y = c_1 e^{3t} + c_2 t e^{3t}$

e.) $y = c_1 e^{-\frac{t}{2}} + c_2 e^{\frac{3t}{2}}$

f.) $y = c_1 \cos\left(\frac{3t}{2}\right) + c_2 \sin\left(\frac{3t}{2}\right)$

g.) $y = c_1 e^{\frac{3t}{2}} + c_2 e^{-\frac{3t}{2}}$

h.) $y = c_1 e^{\frac{t}{2}} + c_2 t e^{\frac{t}{2}}$

3.) a.) $y = e^t$

b.) $y = \frac{5}{2} e^{-t} - \frac{1}{2} e^{-3t}$

c.) $y = 12 e^{\frac{t}{3}} - 8 e^{\frac{t}{2}}$

d.) $y = -e^{t - \frac{\pi}{2}} \sin(2t)$

e.) $y = e^{-2x} \cos(x) + 2e^{-2x} \sin(x)$

f.) $y = 2e^{\frac{2t}{3}} - \frac{7}{3} t e^{\frac{2t}{3}}$

g.) $y = -e^{-\frac{t}{3}} \cos(3t) + \frac{5}{9} e^{-\frac{t}{3}} \sin(3t)$

h.) $y = 2te^{3t}$

i.) $y = -1 - e^{-3t}$

j.) $y = 7e^{-2(t+1)} + 5te^{-2(t+1)}$

4.) a.) $x(t) = c_1 \cos(t) + c_2 \sin(t)$

b.) 2π

c.) $x(t) = c_1 e^{-\frac{\gamma}{2}} \cos\left(\frac{\sqrt{4-\gamma^2}}{2} t\right) + c_2 e^{-\frac{\gamma}{2}} \sin\left(\frac{\sqrt{4-\gamma^2}}{2} t\right)$

d.) $\frac{4\pi}{\sqrt{4-\gamma^2}}$

e.) $\gamma = \frac{\sqrt{20}}{3}$

5.) a.) $0.2Q'' + (3 \times 10^2)Q' + 10^5 Q = 0, \quad Q(0) = 10^{-6}, \quad Q'(0) = I(0) = 0$

b.) $Q(t) = 10^{-6}(2e^{-500t} - e^{-1000t})$

6.) a.) 1.46875

b.) 8.125

7.) a.)

$$\begin{pmatrix} 7 & -3 & 23 \\ 5 & 41 & 47 \\ 20 & 0 & 54 \end{pmatrix}$$

b.)

$$\begin{pmatrix} 26 & -5 & 10 \\ 34 & 46 & 12 \\ 90 & 16 & 30 \end{pmatrix}$$

c.)

$$\begin{pmatrix} 34 & 37 \\ -2 & 32 \end{pmatrix}$$

d.)

$$\begin{pmatrix} -21 & 62 & 51 \\ -41 & 77 & 52 \\ -11 & 17 & 10 \end{pmatrix}$$

e.)

$$\begin{pmatrix} 20 & 30 \\ 32 & 48 \end{pmatrix}$$

f.) 54

g.) 0

h.)

$$\frac{1}{54}\begin{pmatrix} 7 & -1 \\ -2 & 8 \end{pmatrix}$$

i.) Does not exist

j.) $\lambda_1 = 9,\ v_1 = (1,1),\ \lambda_2 = 6,\ v_2 = (1,-2)$

k.) $\lambda_1 = 8,\ v_1 = (1,2),\ \lambda_2 = 0,\ v_2 = (3,-2)$

8.) a.)

$$\vec{x} = c_1 \begin{pmatrix} 2 \\ 3 \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} \tag{1}$$

b.)

$$\vec{x} = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-t} \tag{2}$$

c.)

$$\vec{x} = c_1 \begin{pmatrix} 5\cos(t) \\ 2\cos(t) + \sin(t) \end{pmatrix} + c_2 \begin{pmatrix} 5\sin(t) \\ -\cos(t) + 2\sin(t) \end{pmatrix} \tag{3}$$

d.)

$$\vec{x} = c_1 e^{\frac{t}{2}} \begin{pmatrix} 5\cos\frac{3}{2}t \\ 3\cos\frac{3}{2}t + 3\sin\frac{3}{2}t \end{pmatrix} c_2 e^{\frac{t}{2}} \begin{pmatrix} 5\sin\frac{3}{2}t \\ -3\cos\frac{3}{2}t + 3\sin\frac{3}{2}t \end{pmatrix} \tag{4}$$

e.)

$$\vec{x} = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 \left[\begin{pmatrix} 1 \\ -2 \end{pmatrix} t + \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix} \right] \tag{5}$$

f.)

$$\vec{x} = c_1 e^{-t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 \left[\begin{pmatrix} 2 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} 0 \\ 2 \end{pmatrix} e^{-t} \right] \tag{6}$$

9.) a.)

$$\vec{x} = e^{-t} \begin{pmatrix} -4 \\ -4 \end{pmatrix} + e^{-2t} \begin{pmatrix} 4 \\ 6 \end{pmatrix} \tag{7}$$

b.)

$$\vec{x} = e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{8}$$

c.)

$$\vec{x} = e^{-2t} \begin{pmatrix} \cos(t) - 5\sin(t) \\ -2\cos(t) - 3\sin(t) \end{pmatrix} \tag{9}$$

d.)

$$\vec{x} = e^{-t} \begin{pmatrix} 2\cos(t) + 5\sin(t) \\ -\cos(t) + 12\sin(t) \end{pmatrix} \tag{10}$$

e.)

$$\vec{x} = e^{-t} \begin{pmatrix} 3 \\ -1 \end{pmatrix} - 6te^{-t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{11}$$

f.)

$$\vec{x} = e^{5t} \begin{pmatrix} 2 \\ -5 \end{pmatrix} - te^{5t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} \tag{12}$$

10.) a) Romeo responds to Juliet, Juliet is fickle.

$$\begin{pmatrix} R \\ J \end{pmatrix} = c_1 \begin{pmatrix} \cos(t) \\ -\sin(t) \end{pmatrix} + c_2 \begin{pmatrix} \sin(t) \\ \cos(t) \end{pmatrix} \tag{13}$$

In this case, they will have a neverndinng cycle of love and hate.

b) Romeo and Juliet are 'cautious lovers'. They respond positively to each others's feelings, but strongly inhibit their own if they become too attached.

$$\begin{pmatrix} R \\ J \end{pmatrix} = c_1 e^{-3t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{14}$$

In this case, they will eventually fizzle out to mutual indifference. Too much caution inhibited their relationship

11.) a) Romeo is an eager beaver – excited by Juliet's feelings and responds positively to his own. Juliet finds herself growing attracted to Romeo the more he dislikes her, but remains cautious in her feelings (self-inhibiting)

b) .

$$\begin{pmatrix} R \\ J \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} t \\ 1-t \end{pmatrix} \tag{15}$$

c)

$$\begin{pmatrix} R \\ J \end{pmatrix} = \begin{pmatrix} 1-t \\ t-2 \end{pmatrix} \tag{16}$$

As time goes on, Romeo hates Juliet more and more. Juliet, in turn, begins to love Romeo more and more.

d)

$$\begin{pmatrix} R \\ J \end{pmatrix} = \begin{pmatrix} 2t+1 \\ 1-2t \end{pmatrix} \tag{17}$$

As time goes on, Romeo loves Juliet more and more. Juliet, in turn, begins to hate Romeo more and more.