MA 331 **Name (Print):** _____
Fall 2017
Homework 3: Models of Growth
Due: 09/14/17

1.) Solve the Gompertz equation: Hint: look at the notes from class.

$$\frac{dY}{dt} = rY \ln\left(\frac{K}{Y}\right), \quad Y(0) = y_0$$

2.) Another model of growth is given by the von Bertalanffy equation. Assume the length of a fish, $l(t)$ in mm, at time t (measured in days) is given by the following equation:

$$\frac{dl}{dt} = r(L_\infty - l(t)), \quad l(0) = l_0$$

a.) Solve the equation

b.) What is the asymptotic length of the fish? What does the parameter L_∞ represent?

c.) Assume that the maximal size the fish can attain is 50 mm. The fish has an initial length of 4mm. When the fish is 2 days old, it has a length of 27 mm. Using your equation from a), determine the length of this fish with respect to time. You should have no unknown parameters in your equation (i.e., use the above information to determine r, L_∞ and C.

3.) An extension of the logistic growth equation describing lynx population levels on an island $y(t)$ (measured in hundreds at t years after 2000) is given by :

$$\frac{dy}{dt} = ry\left(1 - \frac{y}{K}\right)\left(\frac{y}{A} - 1\right)$$

Assume $A < K$.

a.) Use the slope field applet to generate the slope field assuming K=10, A=2, and r=5. Draw the following solutions on top of your slope field. $y(0) = 0$, $y(0) = 1$, $y(0) = 3$, $y(0) = 12$. What do you notice?

b.) Draw the phase line to determine the equilibria and their stabilities.

c.) What are the units of parameters K, r, and A. You know the biological interpretations of r and K. Can you determine what the parameter A represents?

4.) Suppose the population of salmon (in thousands) at a farm is described by logistic growth. The salmon farm is trying to determine how to efficiently and sustainably harvest their salmon. The number of fish harvested is proportional to the current population with proportionality constant E(/year). Assume $E < r$

a.) Show the differential equation governing the population of salmon is given by: $\frac{dy}{dt} = ry\left(1 - \frac{y}{K}\right) - Ey$.

b.) Find and classify the equilibrium point(s), y^*

c.) We define the sustainable yield of harvest (number of fish caught) as $H = Ey^*$ where y^* is the stable equilibrium point you found in b. Find the value of E that maximizes the yield and also the maximal yield, H.