MA 331 **Name (Print):** ______________________

Fall 2017

Homework 5: Oscillators & Numerical Methods

Due: 10/12/17

1.) Comparing damped and undamped solutions for the mass-spring system. Assume you have a mass spring system with a mass $m = 10$ kg, spring constant $k = 250$ kg/sec^2, initial position $x(0) = 0.3$ meters, and initial velocity $v(0) = -0.1$ meters/s.

a) Find the solution to this undamped oscillator (Hint: Use the solution derived in class).

b) Now consider a damped system with $\gamma = 60$ kg/sec. Find the solution to the damped oscillator. (Hint: Use the solution derived in class).

c) What effects does damping have on the solution?

2.) Comparing overdamped damped, damped, and critically damped solutions for the mass-spring system:

$$mx''(t) + \gamma x'(t) + kx = 0$$

a) Find the solution to the critically damped oscillator ($\gamma^2 = 4mk$).

b) Let $k = 0.2$ Newtons/meter, $m = 5$ kg, $x_0 = 0.5$ meters, $v_0 = 1$ meters/s. Graph the undamped solution ($\gamma = 0$), the underdamped solution ($\gamma = 1$), the critically damped solution $\gamma = 2$, and the overdamped solution $\gamma = 3$. Turn in a drawing of your solutions. Which case of damping goes to zero fastest?

3.) Consider the differential equation $y' = -2y$, $y(0) = 2$, on the interval [0,4].

a) Use the slope field applet to generate the slope field

b) Solve the differential equation exactly and draw it on your slope field

c) Calculate and plot the solutions you get from Euler's method using $h = 2$ on your slope field. Can you try to explain why the solution is unstable?

4.) Consider the differential equation $y' = y + t^2$, $y(0) = 3$.

a) Show that $y = 5e^t - 2 - t^2 - 2t$ solves the initial value problem

b) Calculate the solution at $t = 1$ using Euler's method with 5 steps (i.e., $h = 0.2$).

c) Calculate the exact solution at $t = 1$. How good was the approximation?

BONUS PROBLEM. In class, we solved the simple harmonic oscillator: $mx'' + kx = 0$, where m represents the mass of an object, k represents the spring constant (force /unit length) and $x(t)$ represents the displacement of the spring from its natural length (unit length), assuming and initial displacement $x(0) = x_0$, and initial velocity $v(0) = v_0$. We got the solution

$$x(t) = x_0 \cos\left(\sqrt{\frac{k}{m}}t\right) + v_0\sqrt{\frac{m}{k}} \sin\left(\sqrt{\frac{k}{m}}t\right)$$

a) Rewrite your solution, letting $w_0 = \sqrt{\frac{k}{m}}$

b) Show that $A\cos(\omega_0 t + \phi) = C\cos(\omega_0 t) + D\sin(\omega_0 t)$, where $A = \sqrt{C^2 + D^2}$, and $\phi = \tan^{-1}\left(-\frac{D}{C}\right)$. (*Hint:* Recall the trigonometric identities: $\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$, $\quad \sin^2(x) + \cos^2(x) = 1$)

c) Rewrite the solution to the simple harmonic oscillator as $x(t) = A\cos(\omega_0 t + \phi)$.