Population Models of Growth

MA 331

Aug 31, 2017

Population Models we've discussed

Exponential growth model

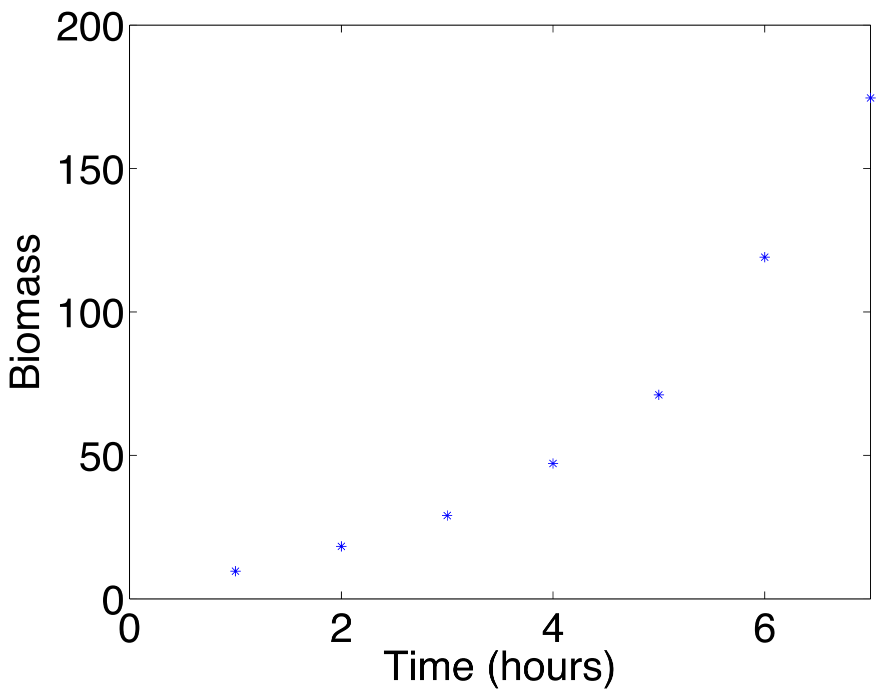
$$\frac{dy}{dx} = ry$$

We call the per capita growth rate:

$$\frac{\frac{dy}{dx}}{y} = r$$

This assumes that per capita growth rate is *constant:* i.e., it does not depend on the current population size, or time

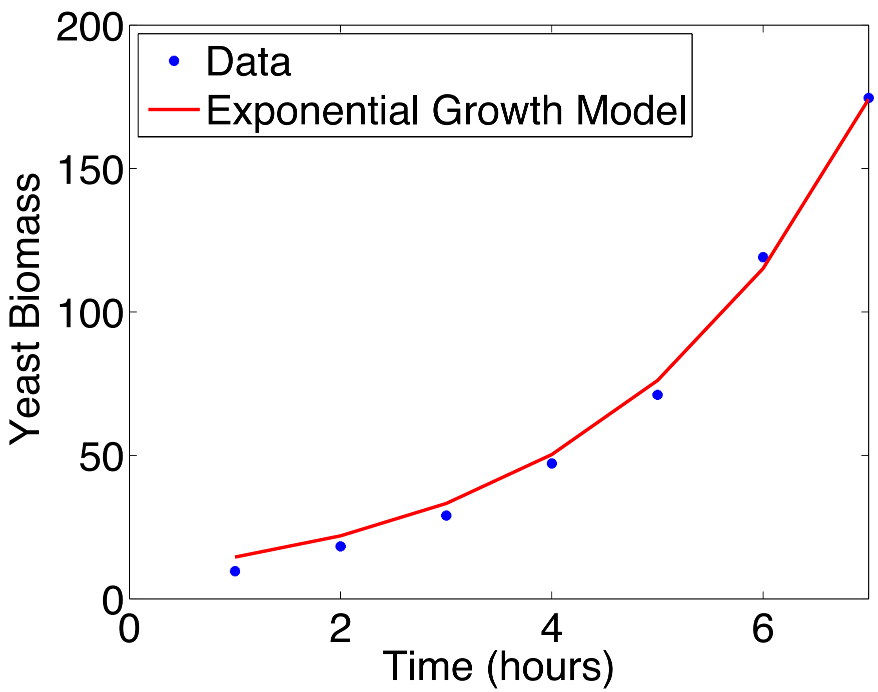
Is this realistic to biological data?



(From Carlson 1913)

This is an experiment where yeast biomass is measured throughout time. It looks pretty exponential

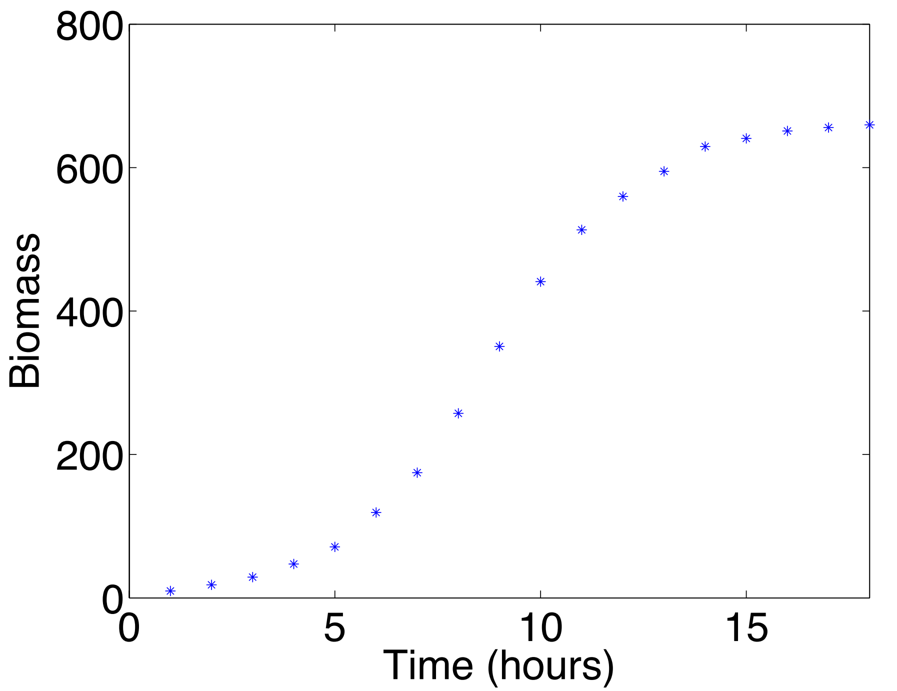
Exponential model looks pretty good…



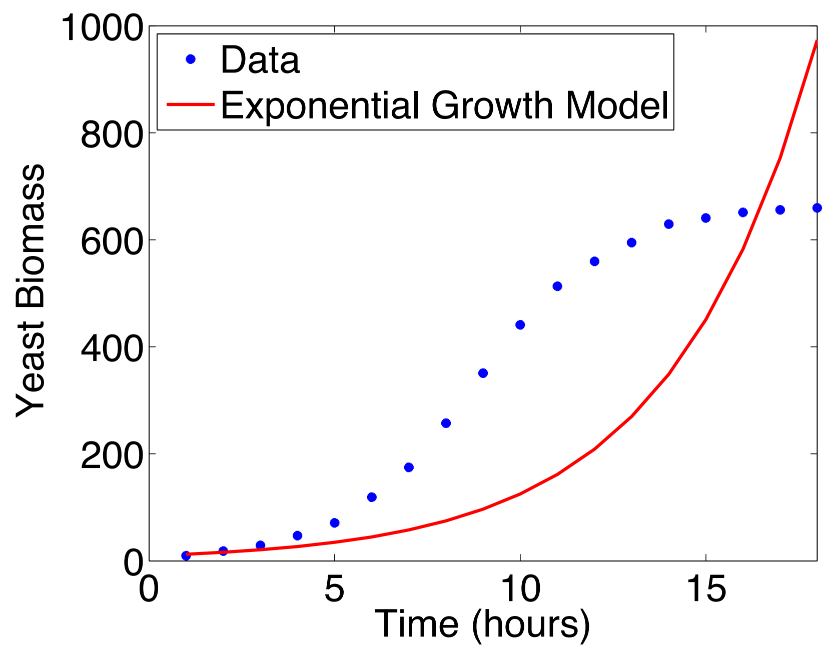
(From Carlson 1913)

We can fit our parameter (the per capita growth rate *r*)

…until we show the full experiment



(From Carlson 1913)



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- Unbounded growth
- Cannot be realistic because there are finite resources (such as food, space, etc)
- What might we expect instead?

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A per-capita growth rate should decrease as the population size Y increases, perhaps becoming zero at some point, or going negative when Y is too large

Logistic Growth

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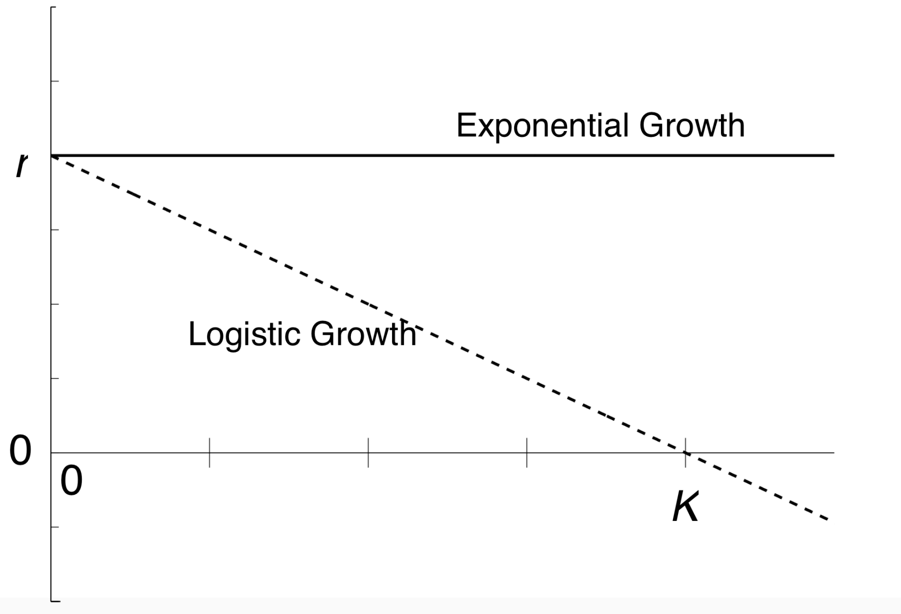
Logistic Growth



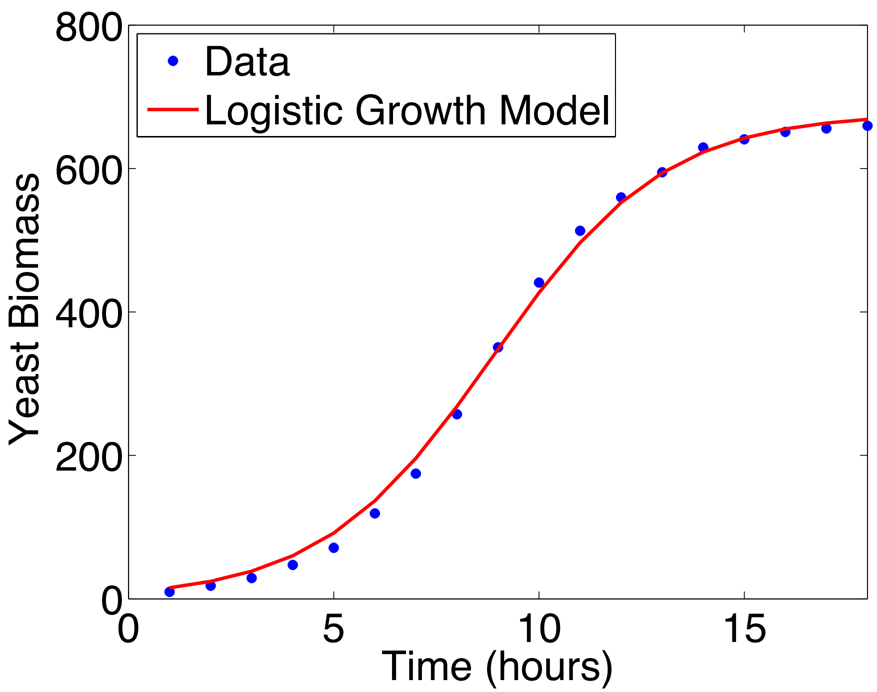
Assume per capita growth decreases linearly with *Y:*

Per-capita growth rate$=r - \frac{r}{K}Y$

$=r\left(1 - \frac{Y}{K}\right)$

K is known as the “carrying capacity”

Logistic Growth fits the model very well!



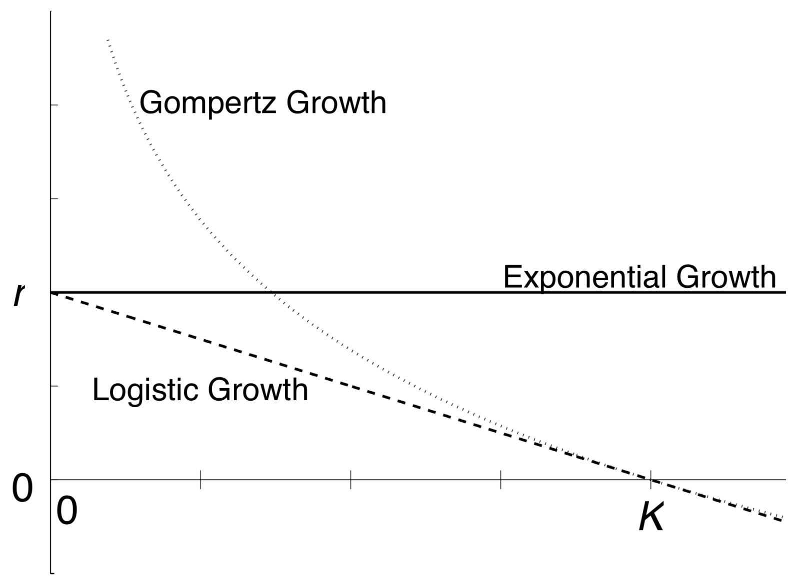
(From Carlson 1913)

We can fit our parameters (the per capita growth rate *r* and the carrying capacity, *K*)

Gompertz Growth

- What is the another way to implement the assumption that per-capita growth rate decreases with increasing population Y and eventually becomes negative?

Gompertz Growth



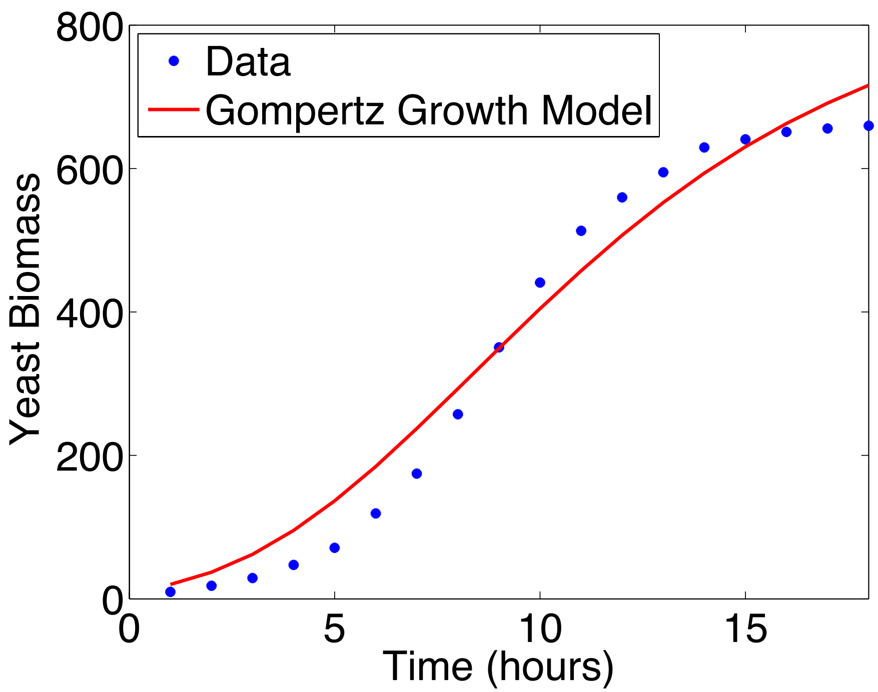
Assume per capita growth decreases logarithmically with *Y:*

Per-capita growth rate$= r \ln\left(\frac{K}{Y}\right)$

K is known as the "carrying capacity"

Allows for unbounded growth for small populations *Y*

Gompertz Growth does not fit the model as well as Logistic Growth



(From Carlson 1913)

We can fit our parameters (the per capita growth rate *r* and the carrying capacity, *K*)

This does not mean the Gompertz growth model is always inferior to logistic growth – only in this particular instance