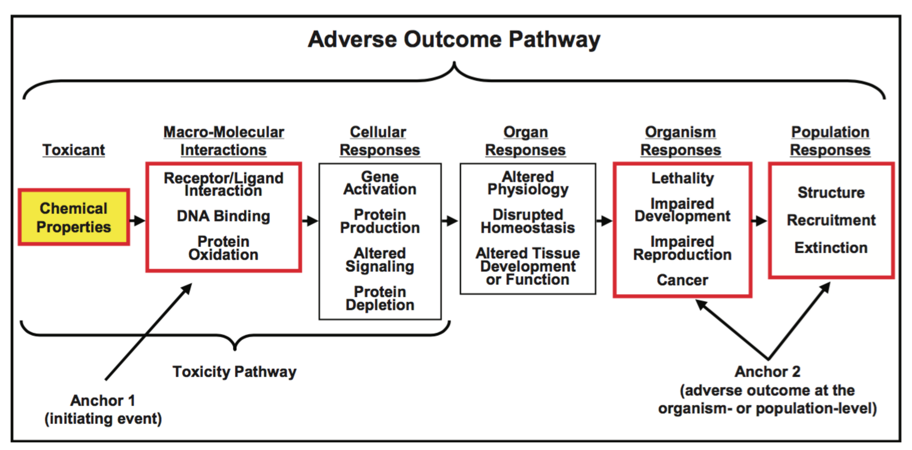
Mathematically Modeling Populations of *Daphnia magna*

Erica M. Rutter, H. T. Banks, G. Leblanc, and K. B. Flores

Center for Research in Scientific Computation,
Department of Mathematics, North Carolina State University

Introduction and Biological Background

- *Daphnia magna* is a species of water flea widely studied in ecotoxicology
- Used to assess hazards of chemicals such as pesticides on ecosystems
- Currently, ecological risk assessments are performed at the organismal level
- Mathematical models are needed to propagate organismal assessment information to population level to enable the causal association of organismal responses to ecosystems adversity (Anchor 2)



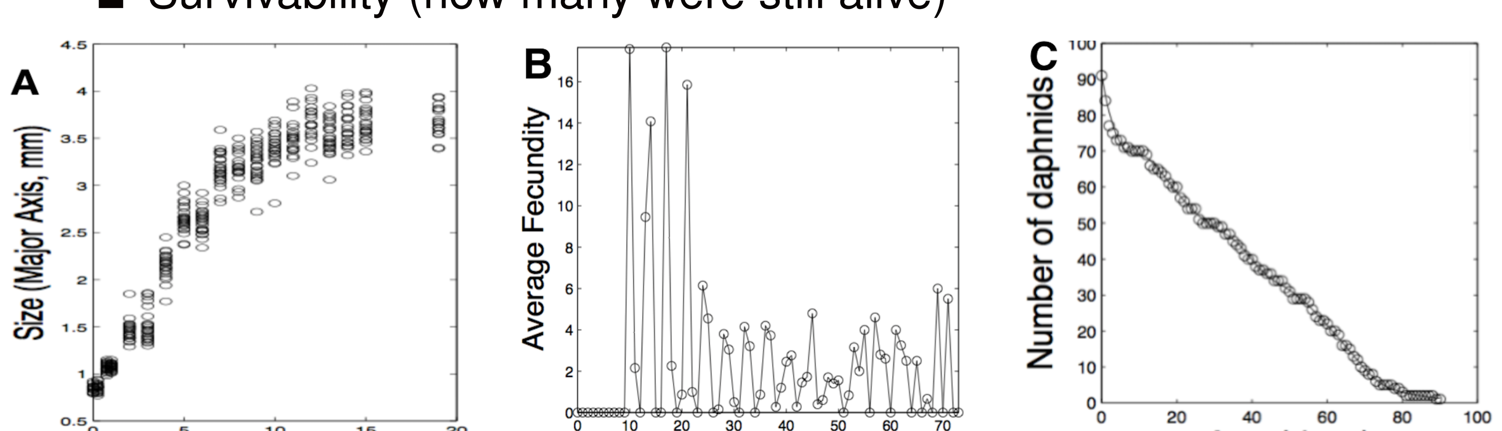
- Biological Questions
  - How do we use individual-level data to inform our population-level predictions?
  - Can we mathematically simulate populations of *Daphnia magna* for over 100 days?
- Mathematical Questions
  - Does our model fit the data well?
  - Do the parameters we find have small confidence intervals and biological meaning?

Data Collection

Individual Level

- 30 individual daphnids are housed in 50 mL beakers with 40 mL of daphnia media
- Daphnids are kept under laboratory conditions (see population level for details)
- Daily, the following were measured:
  - Major axis length, minor axis length (see right)
  - Fecundity (amount of neonates produced)
  - Survivability (how many were still alive)

Example of daphnid with major/minor axis measured. Neonates visible.



Individual-level data collections for growth (A), fecundity (B), and survival (C). These represent density *independent* functions for growth, fecundity, and death.

Data Collection

Population Level

- 2 1-liter beakers are seeded with five 6-day old female daphnids
- Daphnids are kept under laboratory conditions (20 C, 8-16 hour light/dark cycle, media changes daily, 4 mL of 7×10^7 cells/mL algae, *Pseudokirchneriella subcapita*, and 2mL Tetrafin fish food, fed daily)
- Daphnid populations counted every M/W/F for the first 3 weeks, weekly thereafter
  - Daphnids separated by 1.62-mm pore net into size class 1 (less than 1.62 mm) and size class 2 (greater than 1.62mm)

Mathematical Model

- We use the Sinko-Streifer equations that describe continuous-time dynamics of a population structured over the variable age, a
- $u(t,a)$ represents the population of daphnids at time t of age a.

The equation describing daphnid population dynamics is given by:

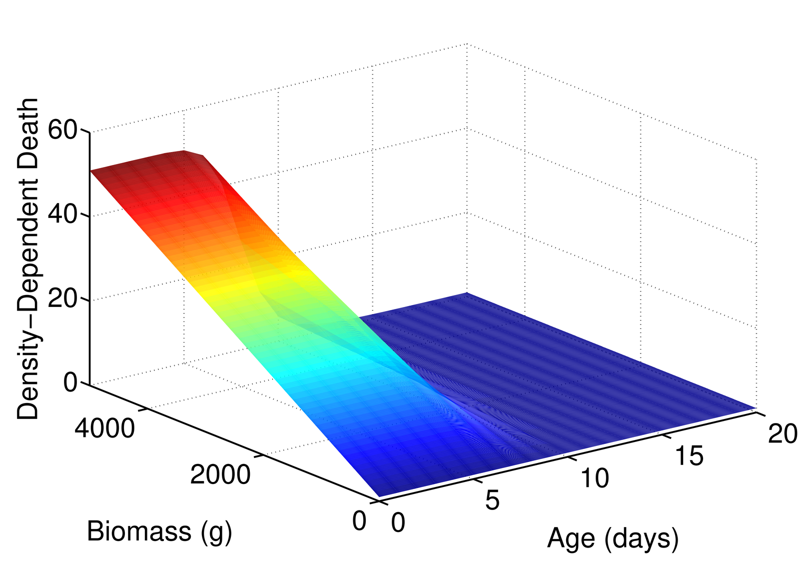
$$\underbrace{\frac{\partial u(t,a)}{\partial t} + \frac{\partial u(t,a)}{\partial a}}_{\text{population change of daphnids}} = - \underbrace{\mu_{ind}(a)}_{\text{density-independent death rate only depends on age, } a \text{ (see figure C bottom left)}} \times \underbrace{\mu_{dep}(a, M(t))}_{\text{density-dependent death rate depends on age and total biomass, } M(t)} \times \underbrace{u(t,a)}_{\text{current population size}}$$

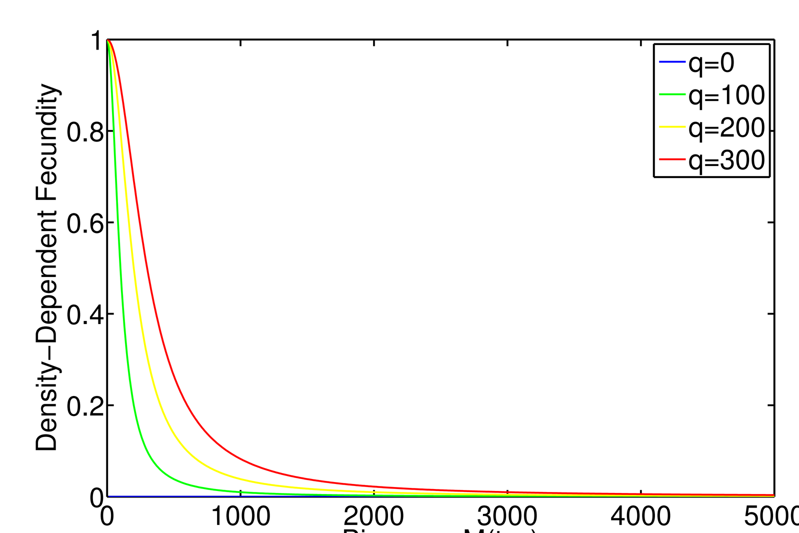
The equation governing the introduction of neonates into the population:

$$\underbrace{u(t,0)}_{\text{neonates being born at time } t} = \int_0^{a_{max}} \underbrace{k_{ind}(s)}_{\text{density-independent fecundity rate depends on age (see figure B bottom left)}} \times \underbrace{k_{dep}(M(t-\tau))}_{\text{density-dependent fecundity rate depends on total population } \tau \text{ days ago}} \times \underbrace{u(t,s)}_{\text{current population size}} \, ds$$

Where total population biomass, $M(t)$ is given by :

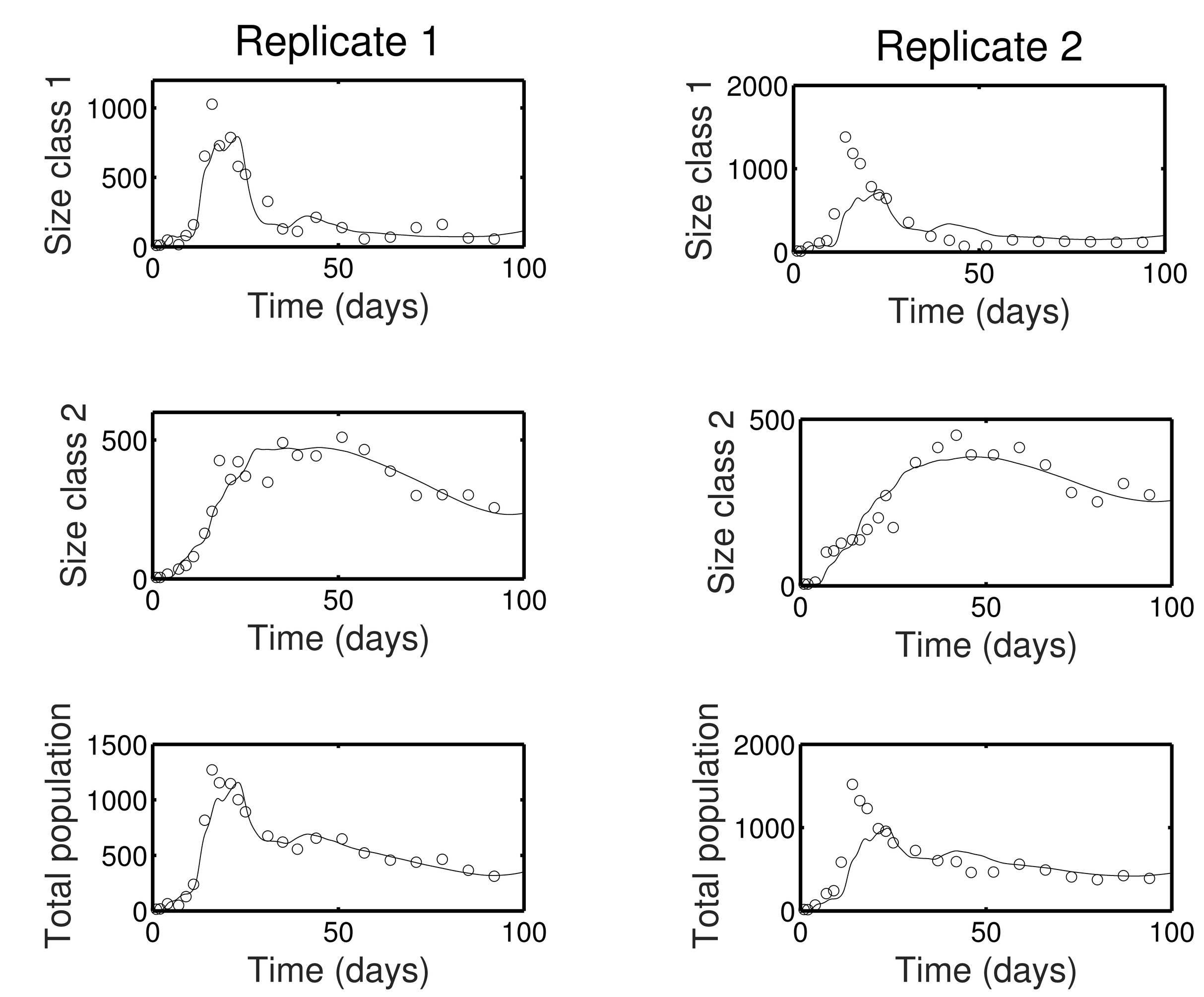
$$\underbrace{M(t)}_{\text{Total daphnid biomass at time } t} = \int_0^{a_{max}} \underbrace{u(t,s)}_{\text{current population size}} \times \underbrace{\left(\frac{K M_0 e^{rs}}{K + M_0(e^{rs}-1)} \right)^L}_{\text{Logistic growth models length of daphnid, (see figure A bottom left) raise to power } L \text{ to get total biomass}} ds.$$





- We estimate 2 parameters by fitting our model to the data
  - q - as shown in the above figure this is responsible for the steepness of the response of density-dependent fecundity
  - c_1 - this represents the linear relationship (steepness) between biomass and density-dependent death

Results



The resulting best fit for our model for replicate 1 (left) and replicate 2 (right). We see the size class one (top, less than 1.62 mm), size class 2 (middle, greater than 1.62 mm) and total population (size class 1 + size class 2)

Parameter	Estimate (Rep1)	95% CI (Rep1)	SE (Rep1)
q	156.8398	(106.7968 , 206.8827)	25.6630
c_1	0.0185	(0.0168, 0.0202)	8.6934e-4
Parameter	Estimate (Rep2)	95% CI (Rep2)	SE (Rep2)
q	245.0448	(108.8946 , 381.1950)	69.8206
c_1	0.0243	(0.0223 , 0.0263)	0.0010

Table: Optimal parameters, confidence intervals, and standard errors for replicates 1 and 2.

Conclusions and Further Directions

Conclusions

- Able to build a realistic population model for *Daphnia magna* and fit to data.
- Used individual-level data to inform population-level microcosm mathematical model.
- Standard errors are small and some parameters are included in the other replicates confidence interval

Further Directions

- Determine why our model underestimates the population peak.
- Test population-level responses to various chemicals/pesticides using previously published individual-level data
- Develop more efficient methods of counting populations
- Use mathematics to optimally design experiments in order to lower standard errors

Funding Acknowledgements: NSF grant number DMS-0946431.