Continuously structured population models for *Daphnia magna*

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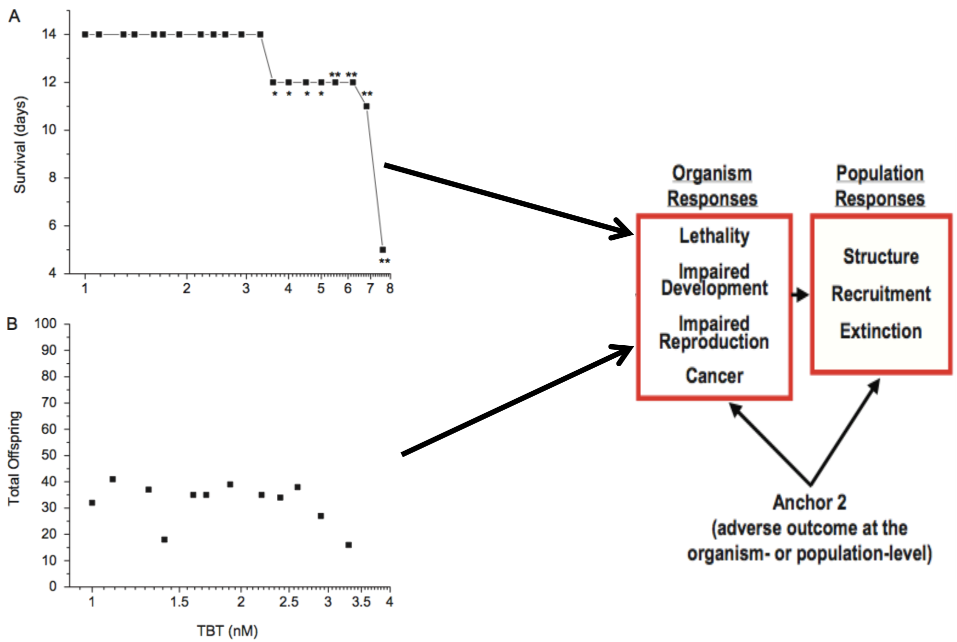
Funding: NSF Math Biology (DMS-1514929), US EPA STAR (RD-835165)

Importance of developing *Daphnia magna* population models

Organism Responses



Wang et al, Tox. Sci., 2011

Structured population models (SPMs)

Model:

$$\underbrace{\frac{\partial u(t,a)}{\partial t} + \frac{\partial u(t,a)}{\partial a}}_{\text{population change of daphnids}} = - \underbrace{\mu_{ind}(a)}_{\substack{\text{density-independent}\\ \text{death rate only}\\ \text{depends on age, } a}} \times \underbrace{\mu_{dep}(a, M(t))}_{\substack{\text{density-dependent}\\ \text{death rate depends}\\ \text{on age and total}\\ \text{biomass, } M(t)}} \times \underbrace{u(t,a)}_{\substack{\text{current}\\ \text{population size}}}$$

$$\underbrace{u(t,0)}_{\substack{\text{neonates being}\\ \text{born at time } t}} = \int_0^{a_{max}} \underbrace{k_{ind}(s)}_{\substack{\text{density-independent}\\ \text{fecundity rate}}} \times \underbrace{k_{dep}(M(t-\tau))}_{\substack{\text{density-dependent}\\ \text{fecundity rate}\\ \text{depends on total population}\\ \tau \text{ days ago}}} \times \underbrace{u(t,s)}_{\substack{\text{current}\\ \text{population size}}} \, ds$$

$$\underbrace{M(t)}_{\substack{\text{Total daphnid biomass}\\ \text{at time } t}} = \int_0^{a_{max}} \underbrace{u(t,s)}_{\substack{\text{current}\\ \text{population size}}} \times \underbrace{\left(\frac{K\, M_0 \epsilon^{r\,s}}{K + M_0(\epsilon^{r\,s} - 1)} \right)^L}_{\text{Logistic growth models length}} \, ds.$$

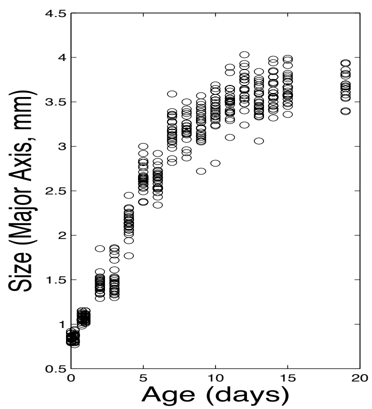
Observables:

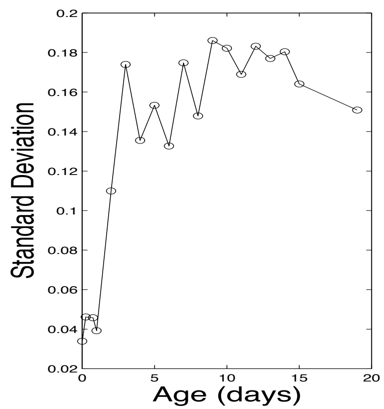
$$S_1(t) = \int_0^{a_1} u(t,s) ds$$

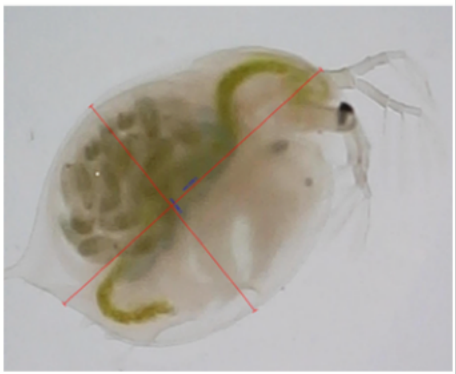
$$S_2(t) = \int_{a_1}^{a_{max}} u(t,s) ds$$

Individual Experimental data

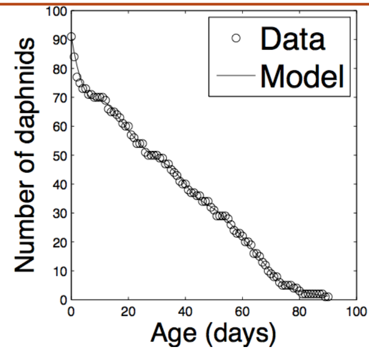
Growth

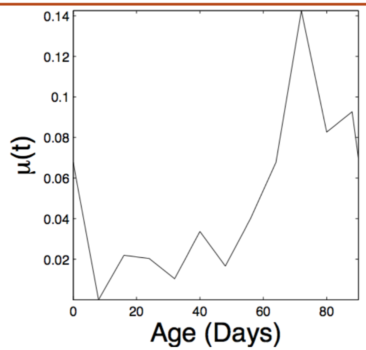






Survival

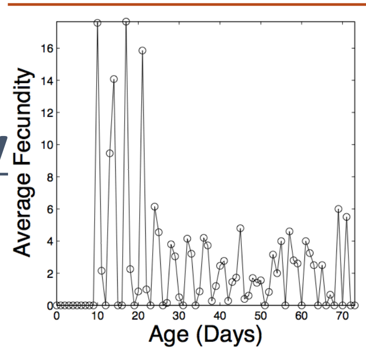


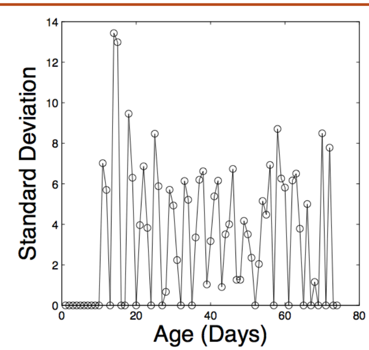


$$\sum_{j=0}^{n} \alpha_j \ell_j(t),$$

$$\ell_j(t) = \begin{cases} \dfrac{t - t_{j-1}}{t_j - t_{j-1}} & \text{if } t \in [t_{j-1}, t_j], \\ \dfrac{t_{j+1} - t}{t_{j+1} - t_j} & \text{if } t \in [t_j, t_{j+1}], \\ 0 & \text{otherwise.} \end{cases}$$

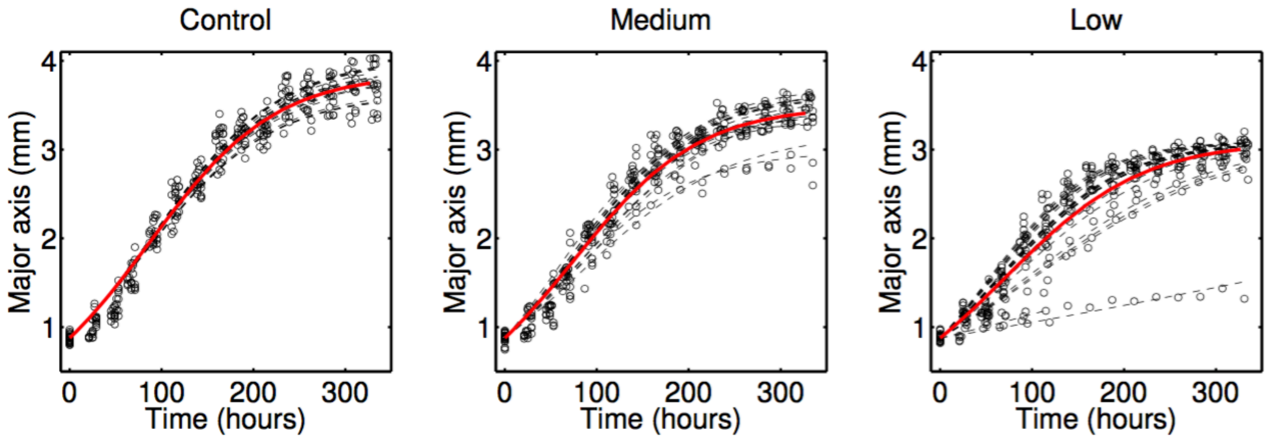
Fecundity





Average daily fecundity was used as the age-dependent fecundity rate.

Daphnid Length vs food availability



Math model for individual growth:

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right)$$

$$x(0) = x_0$$

$$\theta = (x_0, r, K).$$

Hierarchical statistical model:

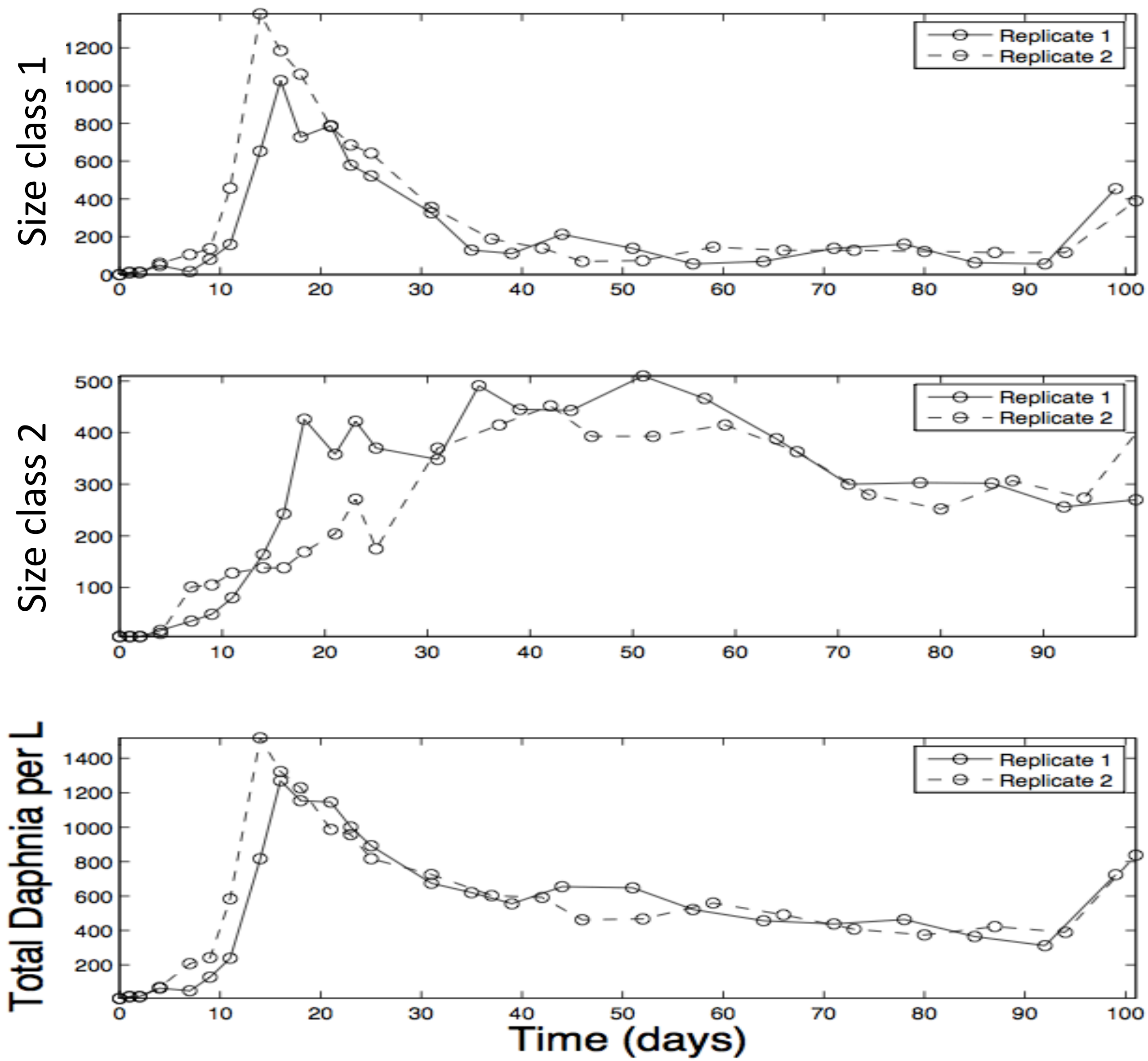
$y_j(t_i) = x(t_i, \theta_j) + \mathcal{E}_{ij}$, with i.i.d. errors $\mathcal{E}_{ij} \sim N(0, \sigma^2)$

for $j = 1, \ldots, D$ individuals at $i = 1, \ldots, P$ time points,

$\theta_j = \beta + \gamma_j$, with "random effects" $\gamma_j \sim N(0, \psi^2)$.

Food Group	Parameter	K	r	M_0
Adoteye et al. [2]	Fixed effect mean value	3.7346	0.0157	.7333
	Random effect variance	0.0010533	0.0048239	6.8978×10^{-7}
Control	Fixed effect mean value	3.8675	0.014398	0.8771
	Random effect variance	0.024703	2.4357×10^{-9}	–
Medium Food	Fixed effect mean value	3.4899	0.014718	0.8771
	Random effect variance	0.036451	2.1267×10^{-6}	–
Low Food	Fixed effect mean value	3.1032	0.013315	0.8771
	Random effect variance	6.6215×10^{-8}	1.5134×10^{-5}	–

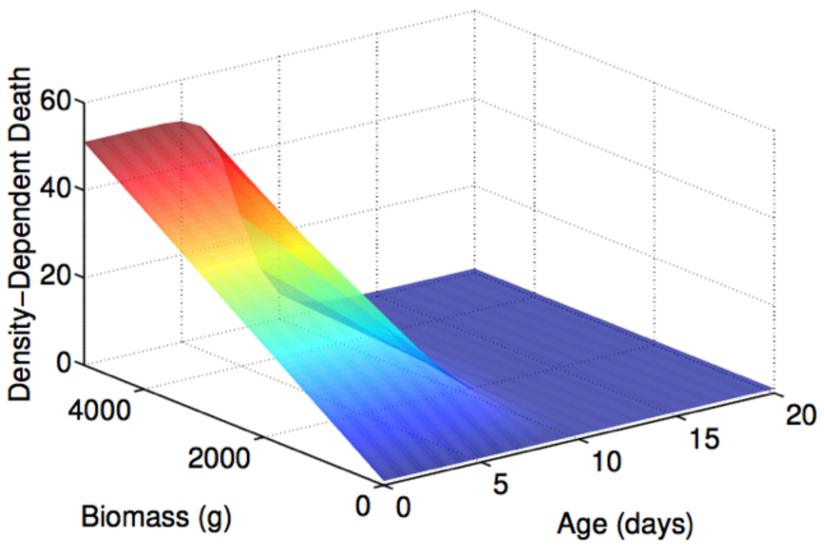
Population Experimental Data

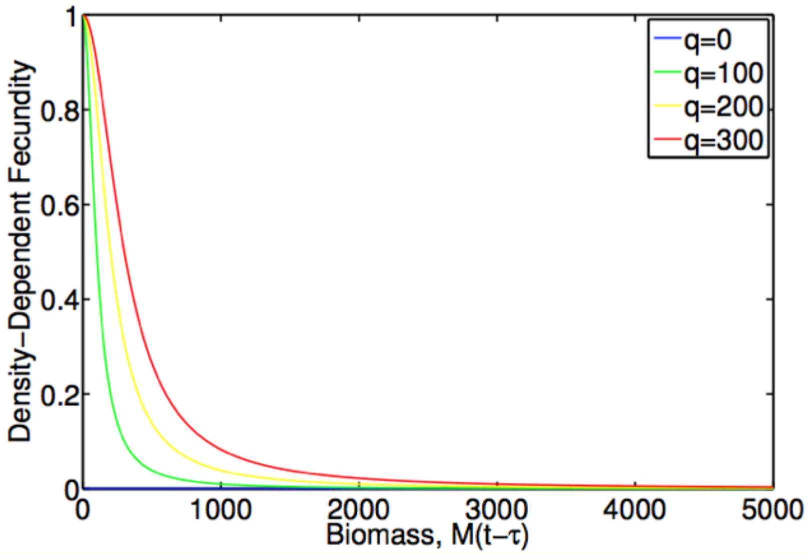


Density-Dependent Functions

$$\mu_{dep}(a, M(t)) = 1 + c_1 M(t) \frac{c_3^{h_2}}{c_3^{h_2} + a^{h_2}}.$$

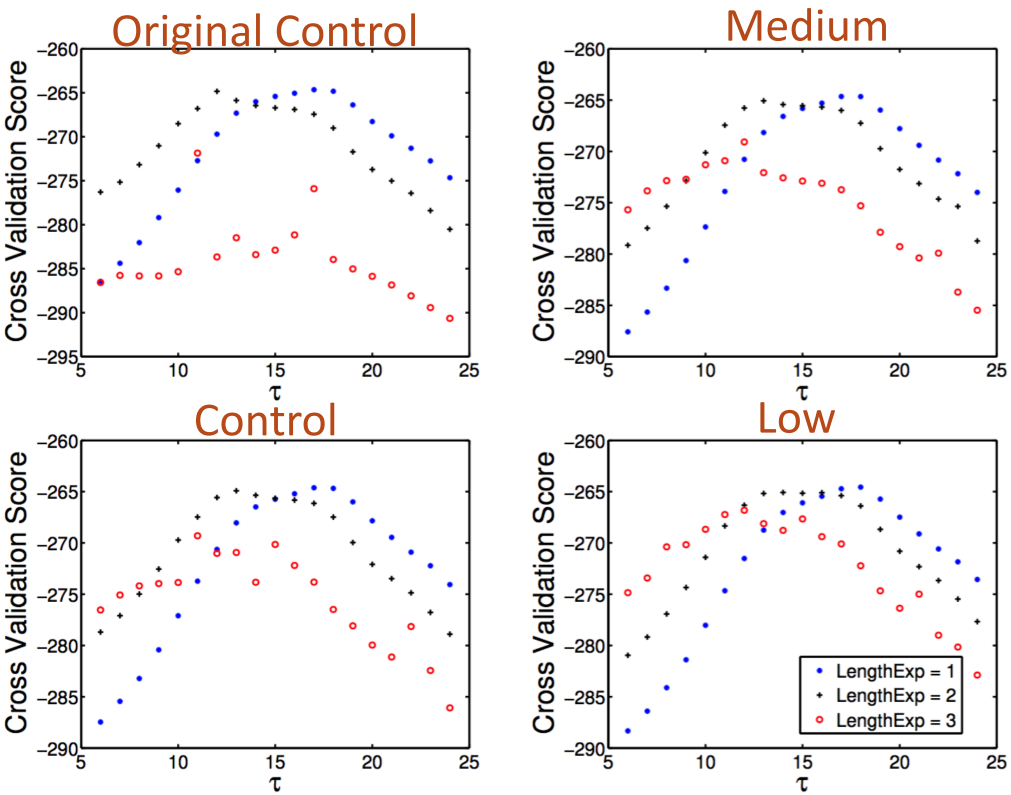
$$k_{dep}(M(t-\tau)) = \frac{q^{h_3}}{q^{h_3} + M(t-\tau)^{h_3}}$$





Hyper-Parameter Selection

Generalizability: L, τ via cross-validation



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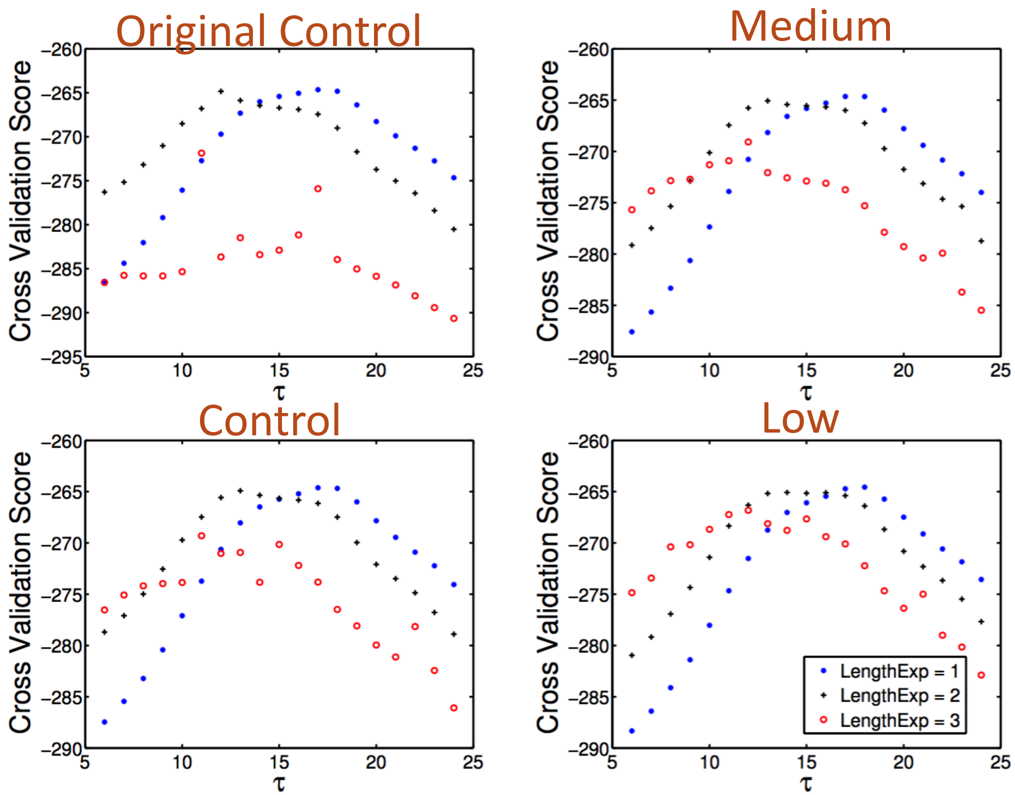
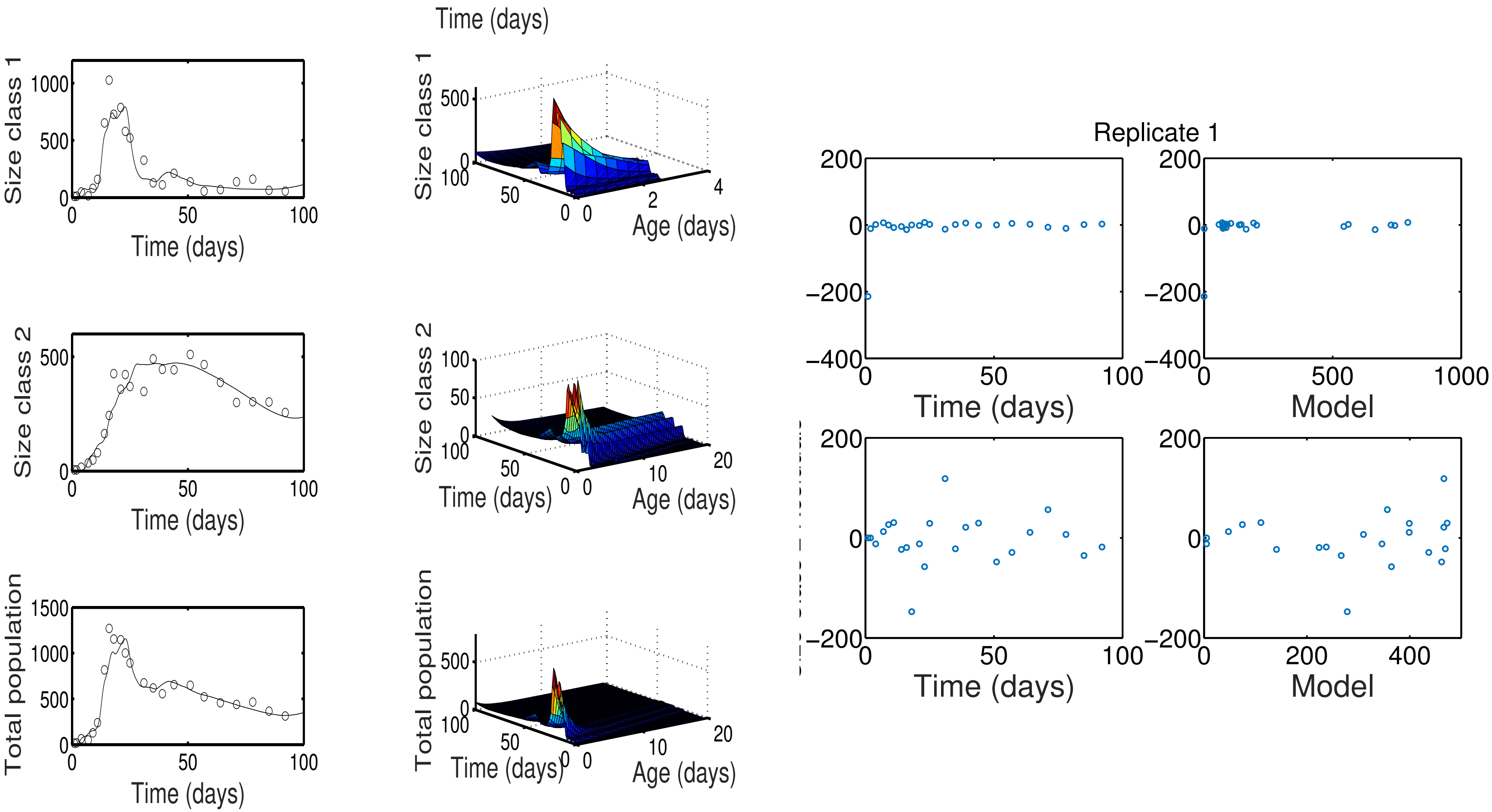


Model Fit: Food Parameters

Food Group	Replicate 1	Replicate 2
Adoteye et al. [2] Food	0.5929	0.2618
Control Food	0.1362	0.2543
Medium Food	0.2558	0.2090
Low Food	0.0151	0.2749

Density-dependent SPM parameter estimation



Rutter et al., Continuous Structured Population Models of *Daphnia Magna. Bulletin of Mathematical Biology, 2017* (accepted).

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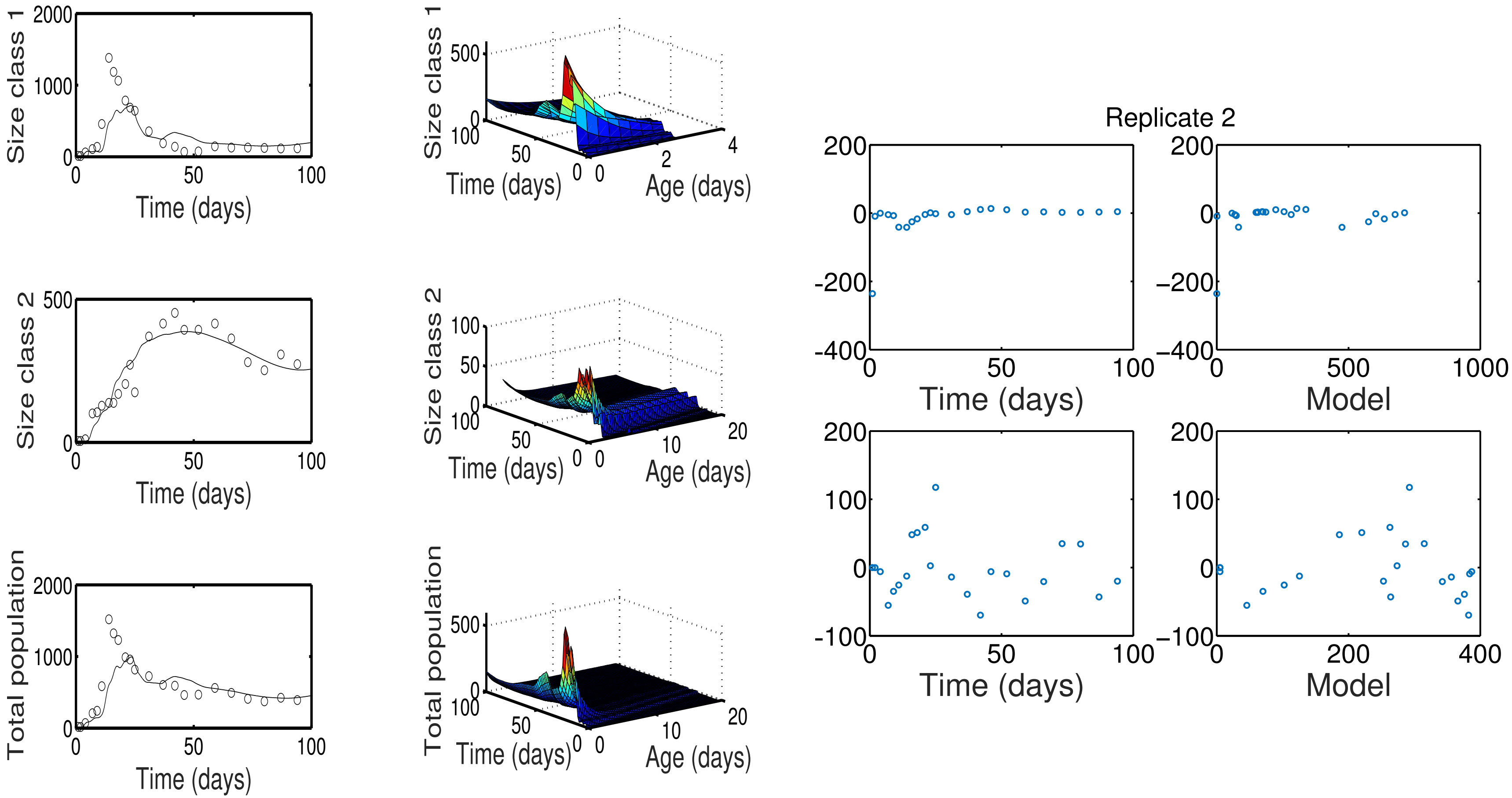

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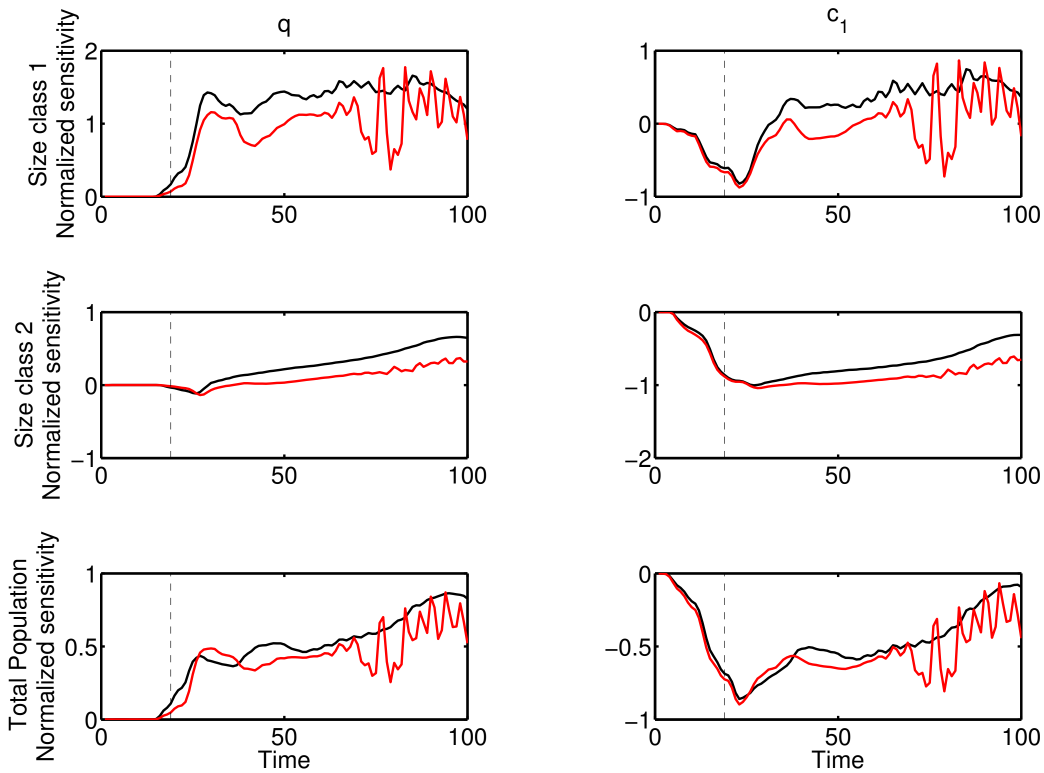
Sensitivity (Rep1 = Black, Rep2 = Red) and CI



Parameter	Estimate (Rep1)	95% CI (Rep1)	SE (Rep1)
q	156.8398	(106.7968 , 206.8827)	25.6630
c_1	0.0185	(0.0168, 0.0202)	8.6934e-4
Parameter	Estimate (Rep2)	95% CI (Rep2)	SE (Rep2)
q	245.0448	(108.8946 , 381.1950)	69.8206
c_1	0.0243	(0.0223 , 0.0263)	0.0010

Data Sparsity Solution 1: Maximizing Information Content

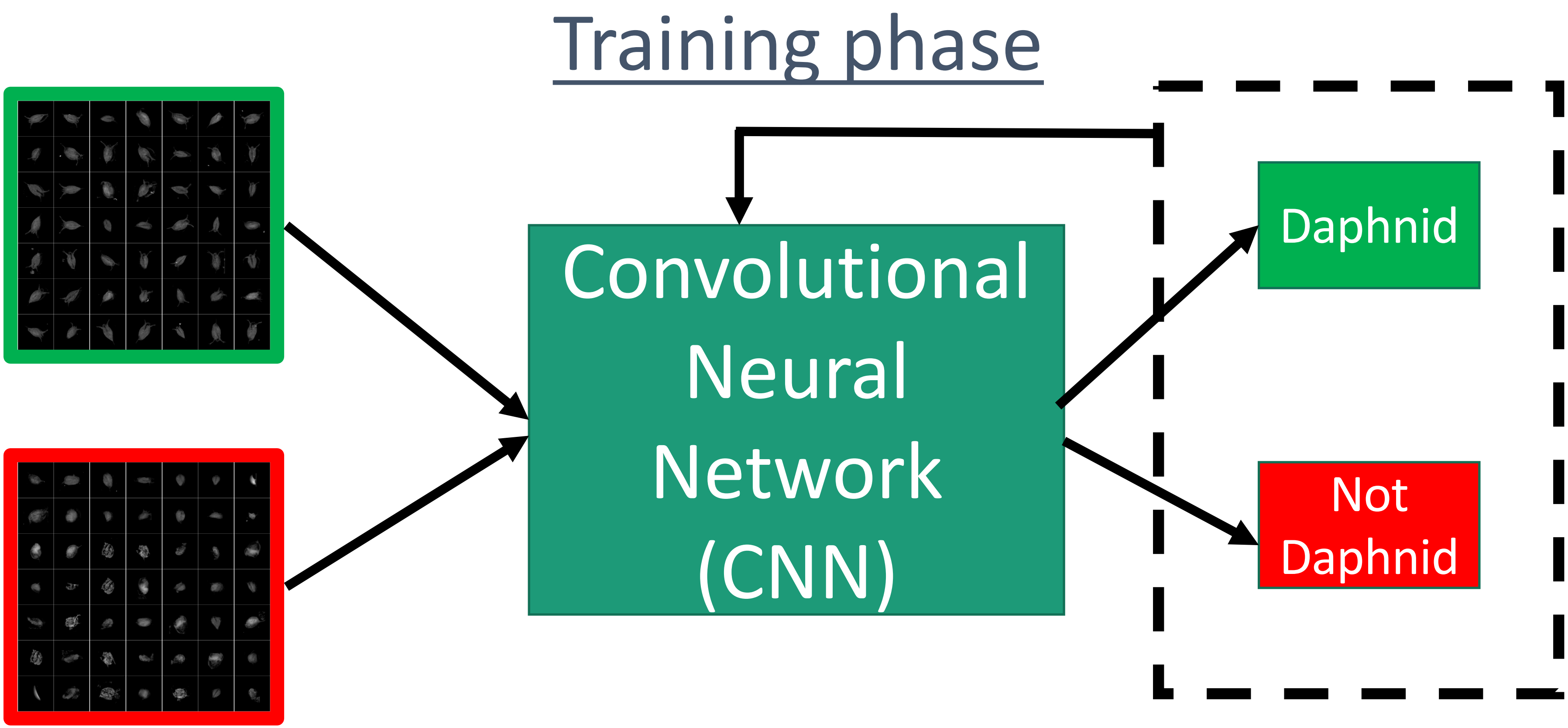
Goal: Intelligently design experiments to maximize information content per data point.

Collection Schedule (# time points)	Replicate 1 SE $q(156.8398)$	Replicate 1 SE $c_1(0.0185)$	Replicate 2 SE $q(245.0448)$	Replicate 2 SE $c_1(0.0243)$
M-F Weekly (67)	13.3679	5.1441e-04	39.8505	5.3772e-04
M/W/F Weekly (40)	17.0327	6.5928e-04	50.7035	6.9229e-04
Tu/Th Weekly (27)	21.5716	8.2271e-04	64.4546	8.5388e-04
M/F Weekly (26)	21.0434	8.0096e-04	64.5870	8.4383-04
Weekly (14)	28.9971	0.0011	92.3124	0.00102
Our Schedule (25)	25.6630	8.6934e-04	69.8206	0.0010

Future Work: Perform optimal design of experiments to minimize SE

Data Sparsity Solution 2: Increasing Efficiency in Data Collection

Goal: Use Machine Learning Algorithms (CNNs) to classify and count images of daphnid microcosm populations



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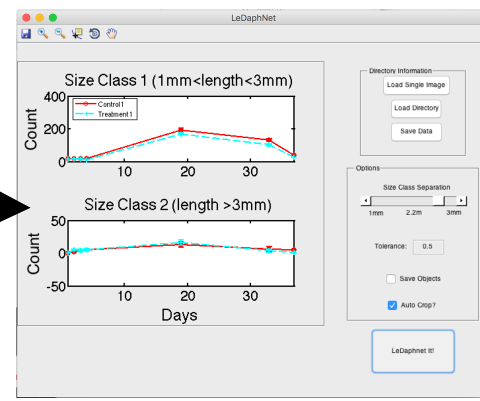

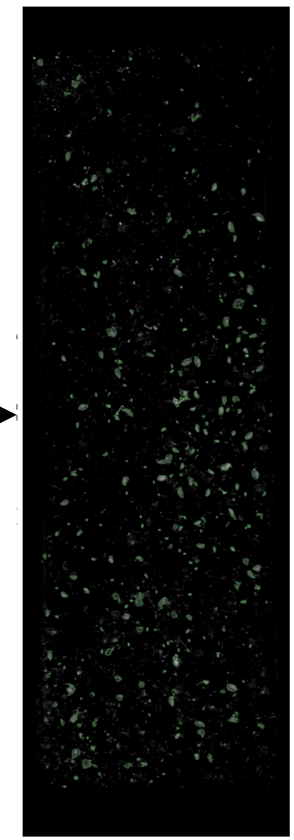

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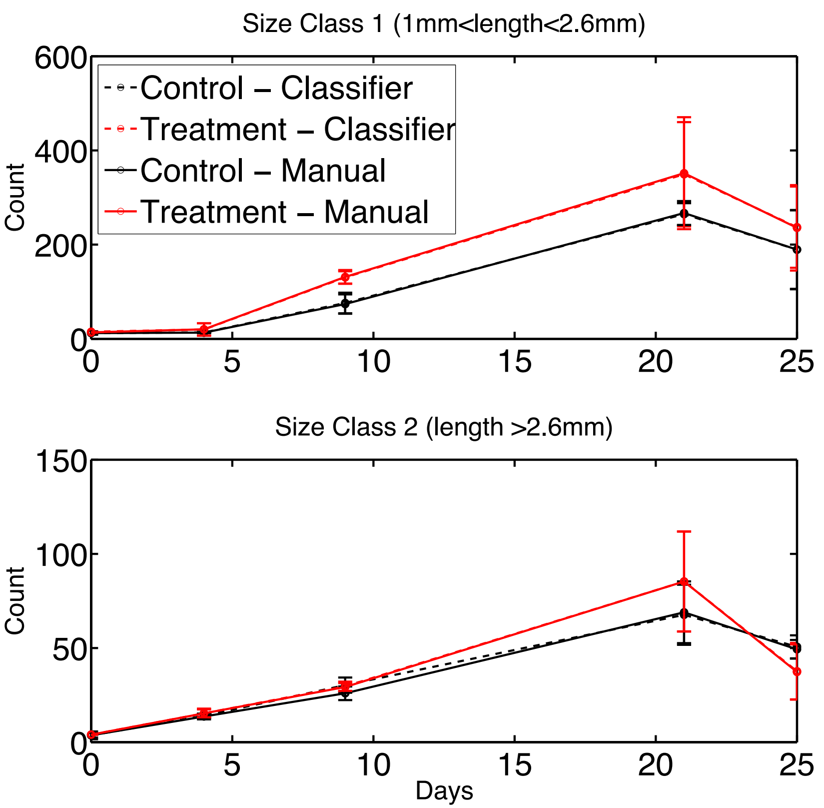

Method	Timing	Accuracy
Hand count with microscope	8 hours	95–100%
Hand count with pipette dropper	70 minutes	90–100%
Scan and manual count	45 mins	95–100%
Subsample and hand count	Not reported	75-88% [3]
LeDaphNet	5 mins	92.5%

Rutter et al., LeDaphNet: An Automated Assessment of *Daphnia magna* Populations Using Digital Imaging and Machine Learning. *Under review.*

Conclusions and Future Directions

- Modeled density-dependent and density-independent size-structured population model for *Daphnia magna*
- Performed parameter estimation, sensitivity analysis and found confidence intervals
- Created a convolutional neural net algorithm to quickly and accurately automatically count populations of *Daphnia magna*
- Optimally design data collection times in order to minimize uncertainty in parameter estimation